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CORRIGENDUM

A high-order spectral method for the study of nonlinear gravity waves

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We have found an error in equations (2.10*a*) and (2.10*b*). We gave these closed-form expressions in passing and without derivation for the special case of shallow-water waves. The correct closed-form expressions for the free-surface evolution equations are:

$$\left.\begin{aligned}\eta_t+\nabla_x\Phi^S\cdot\nabla_x\eta+(1+\nabla_x\eta\cdot\nabla_x\eta)\{\sin[(\eta+h)\nabla_x]\cdot\nabla_x\sec[(\eta+h)\nabla_x]\Phi^S\}&=0,\\ \Phi^S_t+\eta+\tfrac{1}{2}\nabla_x\Phi^S\cdot\nabla_x\Phi^S-\tfrac{1}{2}(1+\nabla_x\eta\cdot\nabla_x\eta)\{\sin[(\eta+h)\nabla_x]\cdot\nabla_x\sec[(\eta+h)\nabla_x]\Phi^S\}^2&=-P_a,\end{aligned}\right\}\tag{C 1}$$

which should replace (2.10) in the original paper. In the above, the sin operator is defined as

$$\sin[(\eta+h)\nabla_x]\cdot\nabla_x\equiv\sum_{n=0}^{\infty}\frac{(-1)^n(\eta+h)^{2n+1}\nabla_x^{2n+2}}{(2n+1)!}.\tag{C 2}$$

The secant operator, which is the inverse of the cosine operator defined in a similar manner to (C 2), can in general be written out, by inspection, to any order of approximation. We give here only the first six terms:

$$\begin{aligned}\sec[(\eta+h)\nabla_x]&=1+f_2+[f_2^2-f_4]+[f_2^3-(f_2f_4+f_4f_2)+f_6]\\&\quad+[f_2^4-(f_2^2f_4+f_2f_4f_2+f_4f_2^2)+(f_2f_6+f_4f_4+f_6f_2)-f_8]\\&\quad+[f_2^5-(f_2^3f_4+f_2^2f_4f_2+f_2f_4f_2^2+f_4f_2^3)+(f_2^2f_6+f_2f_4^2+f_2f_6f_2+f_4f_2f_4+f_4^2f_2+f_6f_2^2)\\&\qquad-(f_2f_8+f_4f_6+f_6f_4+f_8f_2)+f_{10}]+O[(\eta+h)\nabla_x]^{12},\quad|(\eta+h)\nabla_x|<\tfrac{1}{2}\pi,\end{aligned}\tag{C 3}$$

where

$$f_n\equiv\frac{1}{n!}(\eta+h)^n\nabla_x^n.$$

In the special case of constant $\eta+h$, the spatial operators are commutative, (C 3) reduces to the form of the ordinary Taylor expansion of secant, and (C 1) reduces identically to (2.10). As in the original discussion after (2.10), there is an upper bound on the wavenumber (and hence resolution) of the shallow-water approximation, given by $|(\eta+h)\nabla_x|<\frac{1}{2}\pi$, which is the radius of convergence of (C 3).